INDICA: An efficient tool to study the dynamical pantograph-catenary interaction

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1. Introduction

Due to the fact that the railways need to capture adequately the electric power, the force of pantograph-catenary contact has to be maintained as uniform as possible, avoiding the loss of contact. The development of a mathematical model capable of evaluating the mechanical behavior of the system can be helpful in order to obtain optimal assembly conditions in the airline known as catenary contact.

During last years several works have appeared in the scientific literature on the study of pantograph catenary dynamic interaction: in [3] a study based on coupled systems of partial differential equations and algebraic differential equations is presented, [4] presents a simplified method to evaluate the performance of the pantograph, in [5] a procedure based on modal analysis and penalty methods is introduced, in [10] a method using a multibody model and co-simulation is proposed, and finally, [12] presents hybrid procedure using theoretical and experimental modal analysis.

Much of the studies carried out are based on models where the pantograph interacts with a single contact wire along a series of spans of the same characteristics, but this consideration is not completely real, because the catenary is installed in series of 10 or 15 spans, which are not necessarily equal, and wherein the last span of a series and the first span of the next series are overlapped. In the overlapped span, the pantograph can interact with several contact wires at the same time, also presenting a special configuration on the wires in order to obtain a smooth transition between sets of spans. Moreover, in a real assembly, each span can have different characteristics in terms of geometry, number of droppers, etc., so that the identification and generation of the different elements of the differential equation system presents a special difficulty.

In this paper a software tool that allows realistic simulations where several pantographs can interact with the contact wires of two catenaries with overlapped spans, and wherein each span can have different characteristics is presented. A study for the pantograph dynamics from the real model, using independent coordinates and symbolic expressions is also developed.

2. Dynamic equations of the pantograph/catenary system

The pantograph-catenary system is composed of two subsystems interacting with constraint conditions. The dynamic equations for an instant in time t_n, according to the method of Lagrange multipliers is given by:

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \ddot{q}_n \\ \ddot{\lambda}_n \end{pmatrix}+\begin{pmatrix} C_n & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{q}_n \\ \dot{\lambda}_n \end{pmatrix}+\begin{pmatrix} K_n & \phi_n^t \\ \phi & 0 \end{pmatrix}\begin{pmatrix} q_n \\ \lambda_n \end{pmatrix}=\begin{pmatrix} R_n \\ 0 \end{pmatrix}, \quad (1)$$

Where M is the mass matrix, assumed to be constant, C_n is the damping matrix, K_n represents the stiffness matrix, R_n is the vector of external loads on the system, q_n is the vector of generalized coordinates and λ_n is the corresponding vector of constraint forces.

Each catenary in Figure 1 represents a series of spans. When considering two series of spans of the catenary and several pantographs, we must make a partition as on the generalized coordinates, as on the different elements of the differential equation, resulting in the case of two pantographs:

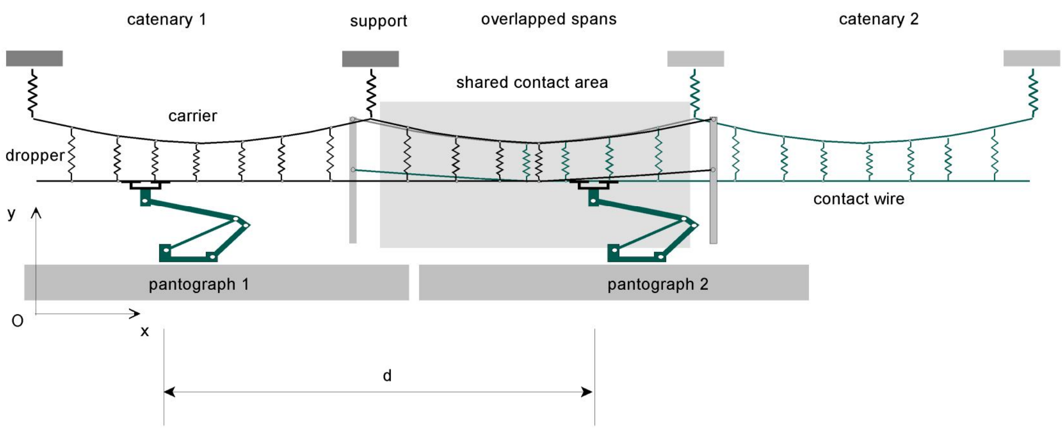

Fig 1. Interaction between two pantographs and two catenaries.

$$q = \begin{pmatrix} q_{C1} \\ q_{C2} \\ q_{P1} \\ q_{P2} \end{pmatrix}, \quad \lambda = \begin{pmatrix} \lambda_{P1} \\ \lambda_{P2} \end{pmatrix}, \quad K = \begin{pmatrix} K_{C1} & 0 & 0 & 0 \\ 0 & K_{C2} & 0 & 0 \\ 0 & 0 & K_{P1} & 0 \\ 0 & 0 & 0 & K_{P2} \end{pmatrix}, \quad R = \begin{pmatrix} R_{C1} \\ R_{C1} \\ R_{P1} \\ R_{P1} \end{pmatrix}, \text{ (2)}$$

Where q_{c1}, q_{c2}, q_{p1}, q_{p2}, represent, in this order, the generalized coordinates of the two catenaries and of the two pantographs. Similar notation can be used for the stiffness matrix and the other terms of equation (1). The λ vector of constraint forces or contact forces is divided into the corresponding terms in the forces on each of the pantographs and on each of the contact wires of the two catenaries.

In order to obtain a model for the cables, we have employed the Finite Element Method, as it is explained in [2] and [6], dividing the cables in a series of segments. The droppers are seen as bars in tension, while the elements of the contact and carrier wire are modeled as a prestressed beam, according to the Euler-Bernoulli equation.

3. Dynamics of the pantograph

3.1. Geometrical study

The pantograph essentially consists of a framework hinged bar of a degree of freedom and a mass of head, with damping and suspension, which can be rotated in the frontal plane and transverse plane of motion, with vertical movement and a total of four degrees of freedom. Some models also consider an intermediate mass with vertical movement and a degree of freedom, to present five degrees of freedom. In the two-dimensional model the frontal rotation is not considered and the transverse rotation is equivalent to suppose the head mass divided into two point masses located either on the intermediate mass or on the articulated frame.

For dynamic simulation purposes, since the bar linkage frame is difficult to model, this element is often simplified and considered as a single point mass with only vertical movement, resulting in the known model of lumped masses, the values of pantograph parameters are generally provided by the manufacturer. In this paper we have developed the dynamic equations of the pantograph, considering the full context bar linkage, as shown in Figure 2. To illustrate more simply the developed model, we have only considered two degrees of freedom: the rotation of the framework and vertical displacement of the head mass. Subsequently the conditions of equivalence between the real and simplified model of lumped masses are established. From the proposed method, the addition of intermediate masses or more movements in the head mass is immediate.

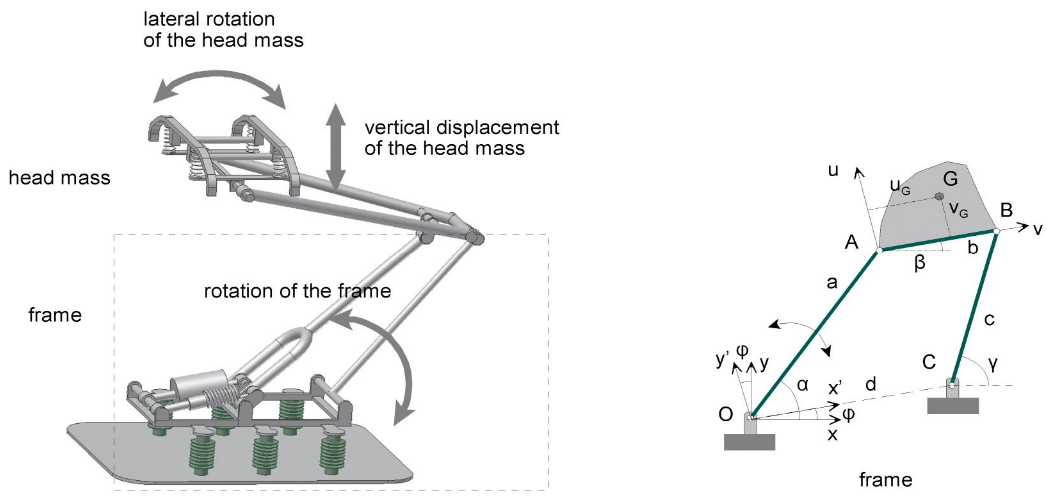

Fig 2. Model of the pantograph.

According to the Figure 2, we can consider that the articulated framework of the pantograph consists of four bars of lengths a, b, c, d linked by joints at points OABC, where the bar OC is fixed. It is possible to express the angles of inclination of the bars AB and BC: β and γ, depending on the angle α of the OA bar, projecting bars on the axes of the fixed system Ox'y', according to Figure 2:

$$\begin{aligned} &a.\cos(\alpha-\varphi)+b.\cos(\beta-\varphi)-c.\cos(\gamma-\varphi)-d=0 \\ &a.\sin(\alpha-\varphi)+b.\sin(\beta-\varphi)-c.\cos(\gamma-\varphi)=0 \end{aligned} \quad (3)$$

The symbolic resolution of this system of equations allows us to express the angles β and γ depending on α. A detailed discussion of this issue, in the case of φ = 0, can be found in [9]. From here, the angular velocities of the bars are:

$$\dot{\beta}=\frac{d\beta}{dt}=\frac{d\beta}{d\alpha}\frac{d\alpha}{dt}=\beta'\dot{\alpha}, \quad \dot{\gamma}=\frac{d\gamma}{dt}=\frac{d\gamma}{d\alpha}\frac{d\alpha}{dt}=\gamma'\dot{\alpha} \quad (4)$$

Also, it is possible to obtain an expression of the position and velocity of any point of the bars depending on the angle α. Let's G the center of gravity of the bar AB, it is possible to obtain the position of G expressed in the Oxy fixed system, from its coordinates $G(u_G,v_G)$ on the local system Auv, linked to the bar:

$$\begin{aligned} x_G &= a\cos\alpha + u_G\cos\beta - v_G\sin\beta \\ y_G &= a\sin\alpha + u_G\sin\beta + v_G\cos\beta \end{aligned} \quad (5)$$

The velocity components of G are obtained by deriving the above expression with respect to time:

$$\begin{aligned} \dot{x}_G &= -\dot{\alpha}a\sin\alpha - \dot{\alpha}\beta' u_G\sin\beta - \dot{\alpha}\beta' v_G\cos\beta = x'_G\dot{\alpha} \\ \dot{y}_G &= \dot{\alpha}a\cos\alpha + \dot{\alpha}\beta' u_G\cos\beta - \dot{\alpha}\beta' v_G\sin\beta = y'_G\dot{\alpha} \end{aligned}, \quad (6)$$

From these expressions, it is also possible to obtain the modulus of the velocity of G, v_G. The generation of the symbolic expressions of the derivatives, can be obtained using MATHEMATICA or MATLAB.

3.2. Kinetic and potential energy

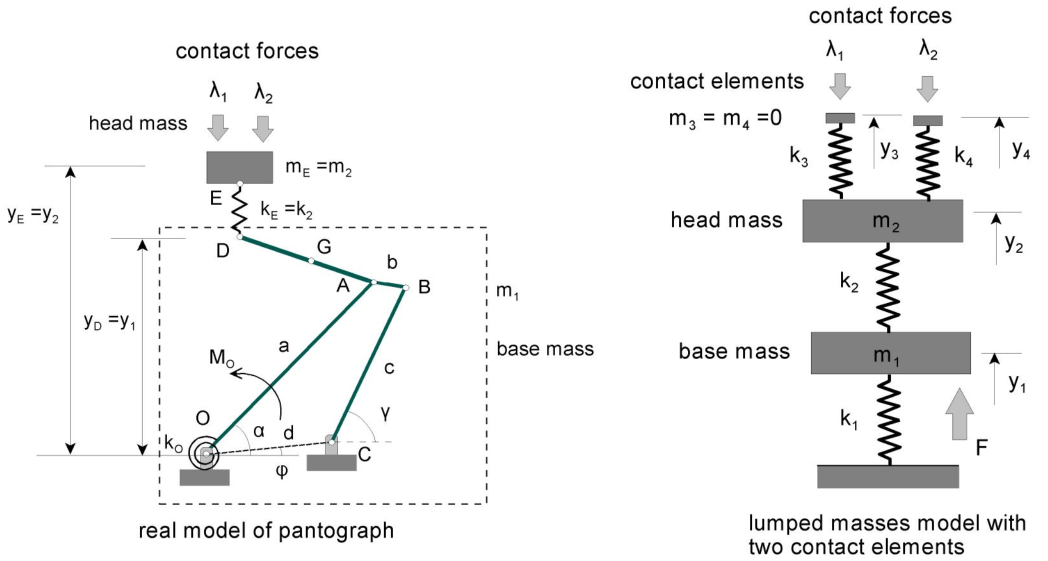

Fig 3. A pantograph of two masses and two contact forces.

It is also possible to express the kinetic energy of the framework, depending of rotation angle of the OA bar, α:

$$T = \frac{1}{2} I_O \dot{\alpha}^2 + \frac{1}{2} I_C \dot{\gamma}^2 + \frac{1}{2} I_G \dot{\beta}^2 + \frac{1}{2} m_B v_G^2 = \\ = 0.5\left[I_O + I_C \gamma'^2 + I_G \beta'^2 + m_B\left(x_G'^2 + y_G'^2\right)\right]\dot{\alpha}^2 = 0.5 H(\alpha)\dot{\alpha}^2 , \quad (7)$$

Where I_O is the moment of inertia of the bar OA on O, I_C the moment of inertia of the BC bar with respect to C, I_G is the moment of inertia of AB with respect to its center of gravity G, and m_B is the mass of bar AB. On the other hand, the frame has a pneumatic cylinder thrust acting on the rod axis OA, which tends to raise the pantograph, to exert a contact force against the catenary, this force can be considered equivalent to a torque M_O on O, while on this point can also be a torsion spring k_O, resulting in a total elastic potential in the pantograph:

$$V = \frac{1}{2} k_O \alpha^2 + \frac{1}{2} k_E \left(y_E - y_D\right)^2 , \quad (8)$$

The position of the point D, in the Figure 3: y_D, can be expressed depending of the angle α, similarly as G on equation (5). From the previous expressions, the Lagrange dynamic equation for the pantograph can be derived, considering α and y_E as independent generalized coordinates. However, to establish and equivalence between the real and the lumped masses models the problem has been restated considering as generalized coordinates the position of D in ABD arm, y_D, instead of the angle α, and the head mass position y_E. The kinetic energy of the framework, under this assumption, is expressed as:

$$T = 0.5H(\alpha)\dot{\alpha}^2 = 0.5\left(H(\alpha)\frac{1}{y_D'^2(\alpha)}\right)\dot{y}_D^2 = 0.5I(\alpha)\dot{y}_D^2, \quad (9)$$

Where y_D and y'_D are known functions of α. I(α) is the coefficient ot the inertia force, being:

$$I(\alpha) = H(\alpha)\frac{1}{y_D'^2(\alpha)} = \frac{I_O + I_C\gamma'^2 + I_G\beta'^2 + m_B\left(x_G'^2 + y_G'^2\right)}{\left(a.\cos\alpha + u_D\beta'\cos\beta - v_D\beta'\sin\beta\right)^2}, \quad (10)$$

The lagrangian of the system will be the kinetic energy less the elastic potential:

$$L = T - V = \frac{1}{2}I(\alpha)\dot{y}_D^2 + \frac{1}{2}m_E\dot{y}_E^2 - \frac{1}{2}k_O\alpha(y_D)^2 - \frac{1}{2}k_E(y_E - y_D)^2, \quad (11)$$

3.3. Dynamic equations

The Lagrange equations for the pantograph are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j, \quad (12)$$

Resulting in a system of two differential equations: one for the equilibrium of the framework and another for the head mass, in particular the dynamic equation of the framework is given by:

$$I(\alpha)\ddot{y}_D + J(\alpha)\dot{y}_D^2 + \frac{k_O}{y_D'(\alpha)}\alpha - k_E(y_E - y_D) = \frac{M_O}{y_D'(\alpha)}, \quad (13)$$

Where the term J(α) corresponds to the coefficient of centrifugal forces. To draw an analogy with the model of lumped masses, a set of approximations in the above equation has to be carried out. The terms of inertia and centrifugal of the dynamic equation (13) vary depending on the angle of rotation α, but in a real situation the pantograph has a configuration that can be considered approximately stable, with minor variations around a central position α_0 , and the parameter I(α) can be considered approximately constant, resulting:

$$m_D = I(\alpha_O) \approx I(\alpha), \quad (14)$$

With respect to the coefficient J(α) of the centrifugal forces, this coefficient has generally very small values and can be neglected. The third term corresponds to the force on the framework of the torsion spring on O. For small oscillations it is possible to linearize the expression in the central value α_O, applying the Taylor rule, finally leading to the equations:

$$\begin{aligned} m_D\ddot{y}_D + k_D y_D - k_E(y_E - y_D) &= F \\ m_E\ddot{y}_E + k_E(y_E - y_D) &= 0 \end{aligned}, \quad (15)$$

Obtaining the expressions of the dynamic equations of the model of lumped masses where F is the driving force equivalent of the pneumatic cylinder, acting on the mass basis. It is possible to add more complexity considering intermediate masses, two head masses, etc, in any case we would obtain dynamics equations that could be expressed in matrix form according to the structure of equation (1).

For simplicity, we have assumed a two lumped masses pantograph. In order to model the contact with two catenaries and overlapped spans two additional contact elements with null mass located on the head, have been considered, as shown in Figure 3, where each contact element touches the contact wire of a different catenary, leading to the mass matrix and the stiffness matrix of the pantograph according to:

$$m_P = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_P = \begin{pmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 + k_4 & -k_3 & -k_4 \\ 0 & -k_3 & k_3 & 0 \\ 0 & -k_4 & 0 & k_4 \end{pmatrix}, \quad (16)$$

In the case of two pantographs, two mass matrices and two stiffness matrices will be considered according to the equation (2).

4. Pantograph/catenary contact model with two contact wires and overlapped spans

The position of the contact elements of the pantograph with the catenary is obtained, according to [2] as a weighted average of the position of the contact wire nodes in the environment of the pantograph. When two catenaries with overlapped spans are considered, several cases in the study of contact can be supposed: the pantograph runs through a normal span of the first catenary, the pantograph runs through a normal span of the second catenary or the pantograph runs through the overlapped spans of both catenaries. In the first two cases, the pantograph interacts at most with a single contact wire, and the position of the contact element will be obtained from any of the following two possibilities, depending on the pantograph is running through the first or second catenary:

$$\left(y_{3,n} = \int_{-l/2}^{l/2} f(x) y_C dx, \quad y_{4,n} = y_{2,n} \right) or \left(y_{3,n} = y_{2,n}, \quad y_{4,n} = \int_{-l/2}^{l/2} f(x) y_C dx \right), \quad (17)$$

Where f(x) is the weighting function, y_C is the position of the contact wire in the environment of the pantograph and l, the length of the zone of friction of the pantograph. In the third case, the pantograph moves through the overlapped spans of both catenaries and can interact with two wires; the position of the contact elements is given by:

$$y_{3,n} = \int_{-l/2}^{l/2} f(x) y_C dx, \quad y_{4,n} = \int_{-l/2}^{l/2} f(x) y_C dx, \quad (18)$$

The above equations also enable set time the matrix $Ø_n$, which corresponds to the conditions of constraint in the dynamic equations (1). It is possible to integrate these equations numerically to simulate the behavior of the system along the time, determining, among other parameters, the variation of the contact forces, the losses of contact, the position of the cables, the position of the contact points, etc. Excellent results have been obtained by explicit method of central differences, according to reference [2].

5. High performance computing approach

In order to solve a mathematical problem in an efficient way on a computer, the following steps are involved [8]:

1. Making a mathematical model of the problem, translating the problem into a mathematical language, eg. ordinary differential equations.
2. Finding or developing constructive methods for solving the mathematical model, that is, a literature search to find what methods are available for the problem.
3. Identifying the best method from a numerical point of view.

4. Implementing on the computer the numerically effective method identified in the previous step.

In general, the developed software has to be high-quality mathematical software which guarantees a good solution to the problem. This high quality mathematical software should have the following features: Power and flexibility, easily read and modified, portability, robustness, efficient and economic in use of storage.

The two last points are especially important in the problem solved in this work. In particular, the sparsity and symmetry of the stiffness matrix has been exploited, improving the efficiency of the implementation and dramatically reducing the memory storage requirements.

Finally, a High Performance Implementation (HPI) has to take into account the features of current architectures like, for example, cache memory. These features are particularly important when rebuilding the traditional algorithms to block-oriented implementations. Block-oriented algorithms reduce drastically the data flow between main memory and secondary memory enhancing the performance of the final implementation. These HPI have been carried out by using BLAS and SPARSKIT standard linear algebra libraries.

The BLAS [7] (Basic Linear Algebra Subroutines) library includes subroutines for common linear computations such as dot-products (BLAS-I), matrix-vector multiplication (BLAS-II), and matrix-matrix multiplication (BLAS-III).

Sparse matrices appear on a lot of current problems in science and engineering. Due to that fact, an intensive research is being carried out in this area producing lot of storage schemes and methods to deal with sparse matrices. SPARSKIT [11] is a software packet which allows us to work with different storage schemes (COO, CSR, CSC, etc) and iterative methods for solving sparse systems of equations. This packet is divided into several modules for conversion of storage scheme (FORMAT module), basic linear algebra operations over sparse matrices (BLASSM and MATVEC module), system of equations solvers (ITSOL module), etc.

6. Some experimental results

In this section, the experimental results obtained with the new HPC implementation of the algorithm for solving dynamical pantograph-catenary interaction.

The test battery used in the experiments is shown in Table 1: Where nv is the number of spans, np is the number of droppers, and lv is the length of the span in meters. For each test, the number of pantographs is varied between 1 and 4. The results, in terms of execution time, are summarizing on Table 2. The properties of the catenary in the test have been taken from [1].

It is possible to appreciate the reduce execution time achieved by using INDICA tool, for solving the pantograph/catenary interaction.

Test	nv	np	lv
1	3	7	20
2	3	17	20
3	3	13	40
4	4	32	60
5	4	40	60

Table 1. Test battery.

Test	1 pant.	2 pant.	3 pant.	4 pant.
1	6.000	6.000	7.000	7.000
2	6.391	6.843	7.328	7.828
3	28.000	29.000	30.000	32.000
4	69.421	72.469	76.218	79.516
5	102.218	107.023	112.861	120.375

Table 2. Execution time (in seconds) for the test battery considered on Table 1.

7. INDICA tool

The algorithm presented in the previous section has been implemented as part of a software tool called INDICA. The software package has been developed on an object-oriented database for the user interface and C language for dynamic and ActiveX libraries for the graphic and visual interface. This framework is supported in the Visual FoxPro (© Microsoft) environment, and it is currently used by ADIF, the Spanish company of railway infrastructures, in the development of its electrical catenary systems. This tool, whose current users interfaces are in Spanish, has a main window control, in with it is possible to choose several options:

- To select several utilities of habitual use, such as cut, copy, exit, about, etc.
- To do the main process, which allows the user to introduce and select the data and the execution of the analysis of the interaction.
- The maintenance of the database system, designed with several files implementing the different tables of a relational database system following a previously designed entity-relation scheme. These tables implement the different auxiliary components that the users can use in the interaction: wire, droppers, pantographs, complements and materials and data for the static structure of the catenary.
- To obtain different reports about the auxiliary databases.
- Some technical utilities for the correct work of the application.

The main procedure in the tool, called "Process", has been defined over a window interface. It allows the input of the different data types, selected among the previously introduced components in the database system, and even some other new data types.

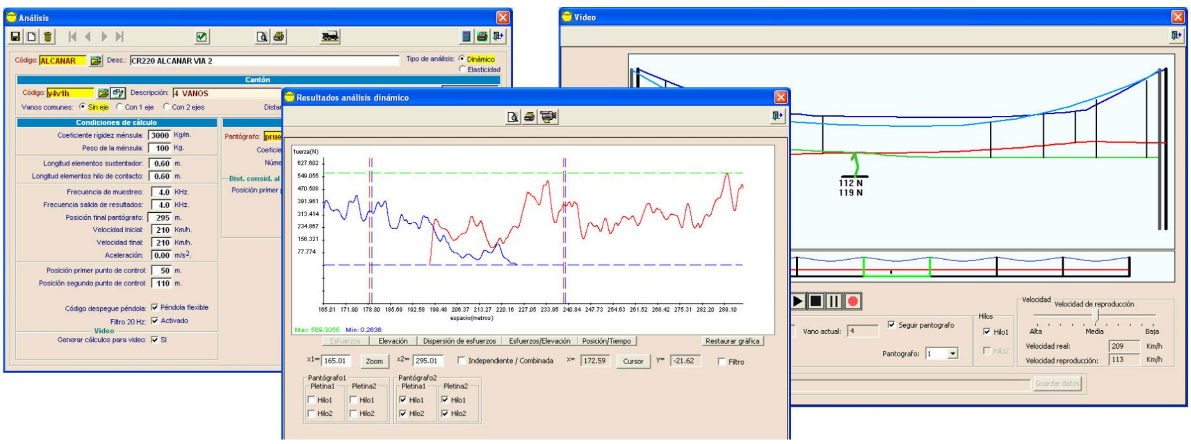

Fig 4. Data Input and presentation of results.

In Fig 4, and example of data inputs over a window of the tool is presented. With the help of a optimal description for any calculation the user is able to locate it again. Further on its parameters can be modified for recomputation.

The design of the different windows in tabulated pages, allows the user to manage the information intuitively and comfortably.

After the user has supplied all the data it is possible to execute de program. This process is short and depending on the complexity of the problem, it can spend some minutes. After that, the user can see, in a new window with tabulates pages, the different obtained results. In Fig. 4 we can see an example of the main windows of the results, in this case, this windows show the picture of the analysis of efforts, and under it some buttons allows obtain other pictures (Fig. 5).

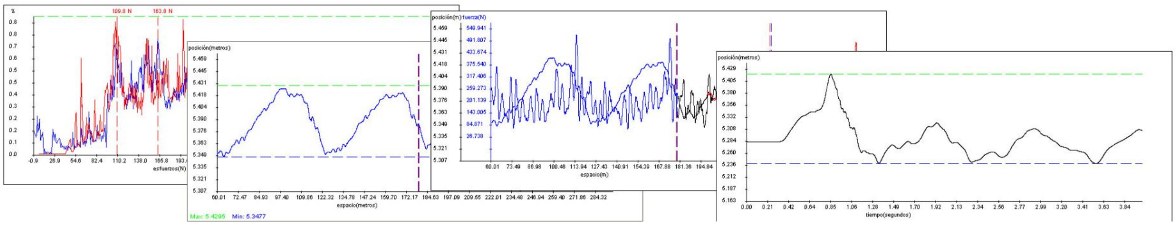

Fig 5. Presentation of results (distribution of forces, elevation, efforts/elevation and position/time).

The obtained results are presented in five possibilities: diagram of efforts, diagram of elevation, diagram of distribution of forces, diagram of relation efforts/elevation and diagram of position/time. Also is possible to obtain a video representing the movement of the pantograph-catenary interaction (Fig 5).

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